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Vibration Reduction in Rotorcraft Using Active Control: A Comparison of Various Approaches

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This paper presents a concise review of the state of the art for vibration reduction in rotorcraft using active controls. The principal approaches to vibration reduction in helicopters described in the paper are 1) higher harmonic control, 2) individual blade control, 3) vibration reduction using an actively controlled flap located on the blade, and 4) active control of structural response. The special attributes of the coupled rotor/flexible fuselage vibration reduction problem are also briefly discussed to emphasize that vibration reduction at the hub is not equivalent to acceleration reduction at specific fuselage locations. Based on the comparison of the various approaches, it appears that the actively controlled flap has remarkable potential for vibration reduction.

Nomenclature

$A_{CX4C}, A_{CX4S}, A_{CY4C}, A_{CY4S}, A_{CZ4C}, A_{CZ4S}$ = 4/rev components of acceleration at fuselage CG
C_W = weight coefficient of helicopter (similar to thrust coefficient)
$F_{HX4C}, F_{HXYS}, F_{HY4C}, F_{HY4S}, F_{HZ4C}, F_{HZ4S}$ = 4/rev components of hub shears
J = quadratic cost function
M_{HCS} = control surface hinge moment
P_{CS} = power required to drive control surface actuators
$[T]$ = higher harmonic control transfer matrix
$\{u_i\}$ = control input for actively controlled flap
$[W_z]$ = diagonal weighting matrix on vibrations
$[W_\theta]$ = diagonal weighting matrix on control amplitudes
$[W_u]$ = diagonal weighting matrix on control input, actively controlled flap
$[W_{\Delta u}]$ = diagonal weighting matrix on rate of control input, actively controlled flap
$\{Z_A\}$ = 4/rev acceleration at fuselage CG
$\{Z_F\}$ = 4/rev hub shear component vector
$\{Z(i)\}$ = vector of vibration amplitudes
$\{Z_0\}$ = vector of baseline vibrations
γ = Lock number
$\delta(\psi_k)$ = flap angle input to kth blade
δ_{nc} = cosine harmonic flap control components
δ_{ns} = sine harmonic flap control components
θ_{HH} = higher harmonic control angle in rotating frame
$\theta_{OS}, \theta_{CS}, \theta_{SS}$ = amplitudes of higher harmonic control sine input in collective, longitudinal, and lateral control degrees of freedom
$\theta_{OC}, \theta_{CC}, \theta_{SC}$ = amplitudes of higher harmonic control cosine input in collective, longitudinal, and lateral degrees of freedom
$\{\theta(i)\}$ = vector of higher harmonic control pitch input
μ = advance ratio
σ = blade solidity
ψ = blade azimuth
ψ_k = kth blade azimuth angle
$\omega_{F1}, \omega_{L1}, \omega_{T1}$ = rotating first flap, lag, and torsional blade frequencies nondimensionalized with respect to Ω
Ω = rotor angular speed
$\bar{\omega}_{HH}$ = higher harmonic control frequency

I. Introduction

VIBRATORY loads in helicopters arise from a variety of sources such as the rotor system, the tail rotor, the engine, and the transmission and lead to fatigue damage of structural components, human discomfort, difficulty in reading instruments, and reduced effectiveness of weapons systems. Consequently, the closely linked problems of vibration prediction and vibration reduction in helicopters have received considerable attention. Excellent reviews on the sources of vibration in helicopters and a description of the methods for reducing vibration levels were presented by Reichert[1] and Loewy.[2]

The central role of vibration reduction in helicopter design has led to the development of two fundamentally different approaches to vibration reduction and alleviation. The traditional approaches that have evolved are passive approaches consisting of vibration absorbers and vibration isolation devices.[1,2] Another passive approach consists of careful structural dynamic design using structural optimization aimed at minimizing vibrations in forward flight.[3] It should also be noted that undesirable vibration levels are often detected during flight testing and the treatment of such problems involves considerable weight penalties.[1,2]

During the last 15 years several active control approaches to vibration reduction have emerged and concise reviews of earlier work in this field can be found in Refs. 4–6. The objectives of this paper are to review research in this area and discuss recent research completed by the first author and his associates. The primary active control approaches to vibration reduction that will be described are 1) higher harmonic control (HHC), 2) individual blade control (IBC), 3) vibration reduction using an actively controlled trailing-edge flap (ACF) located on the blade, and 4) active control of structural response (ACSR)

Each of these approaches will be discussed together with a few illustrative results. Finally, some comments on vibration reduction in coupled rotor/flexible fuselage systems are also made.

II. Higher Harmonic Control

Among the various approaches to active control of vibration HHC, applied through a conventional swashplate (i.e., in the nonrotating reference frame), has received the most attention and therefore represents a fairly mature technology. When employing this method,

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vibration levels in the fuselage or at the hub are reduced by modifying the vibratory aerodynamic loads on the blades. Thus, vibratory forces or loads are modified at their source before they propagate into the airframe. This is in contrast to conventional means of vibration control,[1,2] which attempt to reduce the vibratory loads after they have been generated.

A considerable number of approaches to this problem have emerged since 1967, when Shaw[7] published one of the first treatments of this problem. A particularly successful approach to HHC developed since the early 1980s is based on an adaptive control strategy that combines recursive parameter estimation with linear optimal control theory. A class of such algorithms has been discussed in detail by Johnson[6] and was also analytically studied by Molusis et al.[8] Wind tunnel tests,[9] digital simulations,[10–14] and flight tests of an OH-6A helicopter[15] on which this particular control algorithm, denoted as the "cautious controller," was implemented have demonstrated the validity of this approach for producing very substantial reduction of vibration levels in forward flight.

This body of research has had a major impact on the field since all HHC systems flight tested after the initial OH-6A flight test have employed a conceptually similar control algorithm.[16–19] Therefore, this research and subsequent research related to it will be described next. The majority of these HHC studies, either analytical or experimental,[6,8–19] have been based on linear, quasistatic, frequency-domain representations of the helicopter response to control. Least squares or Kalman filter type identification of helicopter control parameters has been used together with a minimum-variance or quadratic performance function type controller to determine the optimal control harmonics for vibration alleviation. A detailed description of the control algorithm used in these studies can be found in Refs. 6, 8, 14, 20, and 21. In these studies, the general HHC input is expressed as

$$\begin{aligned}\theta_{HH} &= (\theta_{OS}\sin\bar{\omega}_{HH}\psi + \theta_{OC}\cos\bar{\omega}_{HH}\psi)\\ &\quad + (\theta_{CS}\sin\bar{\omega}_{HH}\psi + \theta_{CC}\cos\bar{\omega}_{HH}\psi)\cos\psi\\ &\quad + (\theta_{SS}\sin\bar{\omega}_{HH}\psi + \theta_{SC}\cos\bar{\omega}_{HH}\psi)\sin\psi\end{aligned}\tag{1}$$

where $\theta_{OC}, \theta_{OS}, \theta_{CS}, \theta_{CC}, \theta_{SS}$, and θ_{SC} are independent of ψ.

Minimum-variance controllers are obtained by minimizing the cost functional

$$\begin{aligned}J = E\big(&\{Z(i)\}^T[W_Z]\{Z(i)\} + \{\theta(i)\}^T[W_\theta]\{\theta(i)\}\\ &+ \{\Delta\theta(i)\}^T[W_{\Delta\theta}]\{\Delta\theta(i)\}\big)\end{aligned}\tag{2}$$

where $\Delta\theta(i) = \theta(i) - \theta(i-1)$. Typically $\{Z(i)\}$ and $\{\theta(i)\}$ consist of the sine and cosine components of the N-per-revolution vibrations and HHC inputs, respectively, during the ith control step. The index i on both the vibrations and control input reflect the discrete-time nature of control strategies based on frequency-domain representations of the helicopter response to control. The sampling time step Δt must be sufficient for all transients to die out and for vibration harmonics to be measured. This time step would typically be one rotor revolution.[6]

The weighting matrices $[W_Z]$, $[W_\theta]$, and $[W_{\Delta\theta}]$ on the vibrations, control, and rate of change of control, respectively, may be changed to modify the relative importance of these various components.

The minimum-variance controllers are obtained by taking the partial derivative of J with respect to $\{\theta(i)\}$ and requiring

$$\frac{\partial J}{\partial\{\theta(i)\}} = \{0\}\tag{3}$$

The resulting set of equations may be solved for the optimal HHC input denoted by $\{\theta^*(i)\}$.

The form of the final algorithm depends on the modeling of the helicopter response to control. Two linear, quasistatic, frequency-domain representations of the helicopter response to HHC are frequently used.[6]

The global model of helicopter response to HHC is based on assuming linearity over the entire range of control application:

$$\{Z(i+1)\} = \{Z_0\} + [T]\{\theta(i+1)\}\tag{4}$$

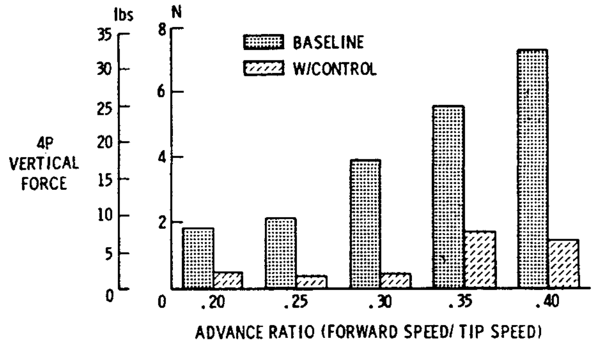

Fig. 1 Variation of vibratory vertical force with advance ratio using adaptive HHC in wind tunnel test.[9]

The vibration vector $\{Z\}$ at step $i+1$ is equal to the baseline uncontrolled vibration level $\{Z_0\}$ plus the product of the transfer matrix $[T]$ and the control vector $\{\theta\}$ at step $i+1$. This implies that $[T]$, which is the transfer matrix relating HHC inputs to vibration outputs, is independent of $\{\theta\}$.

The local model of helicopter response to HHC is a linearization of the response about the response to the current value of the control vector:

$$\{Z(i+1)\} = \{Z(i)\} + [T](\{\theta(i+1)\} - \{\theta(i)\})\tag{5}$$

or

$$\{\Delta Z(i+1)\} = [T]\{\Delta\theta(i+1)\}\tag{6}$$

which implies that the transfer matrix $[T]$ varies with the input $\{\theta\}$.

Each of these two algorithms has two versions, deterministic and cautious; this depends on the assumptions made on the noise characteristics for each row of $\{Z\}$ and $[T]$.

Another ingredient in this algorithm is associated with identification. In applying HHC algorithms to vibration reduction, it is assumed that the HHC inputs $\{\theta(i)\}$ are known without error. Based on the measurements, different parameters may be identified. For the global model, the baseline vibration vector $\{Z_0\}$ may be identified. For the local model, the transfer matrix $[T]$ must be identified. The general discrete Kalman filter is frequently used in the identification process.[6,8,14,20]

Hammond[9] conducted extensive wind tunnel tests on an aeroelastically scaled, four-bladed, articulated helicopter rotor model. A number of alternative algorithms were tested, and it was found that the cautious controller gave very good performance. A typical result[9] showing the variation of the vibratory vertical force with advance ratio is presented in Fig. 1. Reduction between 70% and 90% for this vibratory component was obtained over the range of advance ratios tested. The results also indicated that HHC inputs produce increased edgewise bending moments, torsional moments and control loads. The increased loads experienced during the tests were within the design loads. This wind tunnel test was intended to support a subsequent full-scale flight demonstration test of an OH-6A helicopter equipped with a HHC system. The results of the full-scale tests were reported in a detailed paper.[15] The aircraft was flown from zero airspeed to 100 knots, with the HHC system operated in both the open-loop mode (manually) and the closed-loop mode (computer controlled). Flight tests results exhibited significant reduction in helicopter vibrations without undue penalties in blade loads and aircraft performance. Figure 2, taken from Ref. 15, shows the vertical acceleration at the pilot seat as a function of airspeed when the closed-loop HHC-4P is operating and when it is turned off. The reductions in vibration levels are remarkable.

Flight tests of an experimental HHC system on an SA349 Gazelle were also conducted in France.[18,19] A detailed description of both the simulations and the flight tests were presented in Refs. 18 and 19. The control algorithms are similar to those used previously,[8,14] and a reduction of 80% in cabin vibrations at an airspeed of 250 km/h was demonstrated.

It is remarkable that a significant portion of the research in the field of HHC for vibration reduction involved primarily wind tunnel

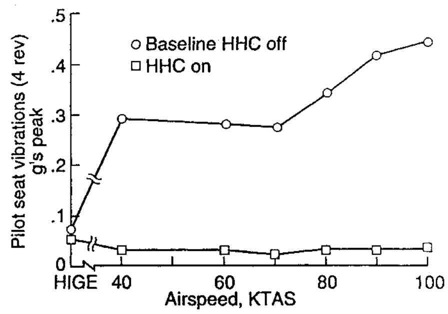

Fig. 2 Vertical vibration reduction at pilot seat.[15]

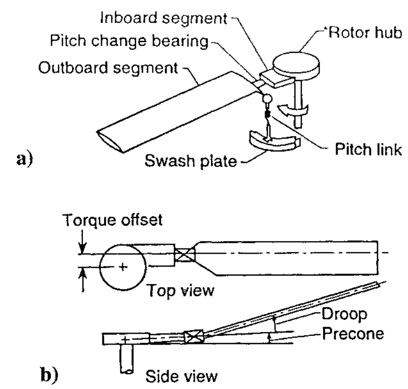

Fig. 3 Typical hingeless rotor blade geometry.

testing[9,22] and flight testing.[15–19] Although a number of an analytical studies were performed,[8,10,14,23–25] some were less comprehensive than the tests. Previous analytical studies have generally relied on simple analog[25] or frequency-domain[24,25] models of helicopter response. The helicopter aeroelastic problem is inherently nonlinear, with design parameters interacting in a complex manner. Thus simplistic models of helicopter response can be unreliable. Other studies[11,12,14] were based on a fairly old aeroelastic response code, the G4000,[26,27] for simulation purposes.

Recently a comprehensive aeroelastic simulation capability has been developed[20,21,28,60] and used to study a number of fundamental issues in HHC. The analysis is based on a coupled flap-lag-torsional blade model in forward flight, with time-domain unsteady aerodynamics and completely coupled aeroelastic response and trim analysis. The response analysis is based on three-flap, two-lag, and the fundamental torsional mode. The four-bladed rotor is assumed to be attached to a fixed, rigid fuselage; thus only hub shears and moments are simulated analytically. The HHC input is represented by Eq. (1). A deterministic and cautious minimum-variance controller was programmed into two algorithms, one for local and one for global HHC models.[6,14,20]

The typical hingeless rotor blade considered in Refs. 20, 21, 28, and 60 is shown in Fig. 3. Using this model various aspects of the HHC implementation on a soft-in-plane hingeless rotor were carefully studied. A few useful results and conclusions obtained in these studies are briefly summarized below.

It was found that both local and global controllers performed well and were effective in reducing the vibration levels in a four-bladed soft-in-plane hingeless rotor. Deterministic and cautious versions of these two controllers were equally successful in reducing the 4/rev vibratory hub shears from their baseline values calculated in the absence of HHC input.

The controllers were also studied under transient flight conditions as represented by a 15% step change in forward flight velocity, and their performance was found to be satisfactory. The local controller exhibited a slightly more oscillatory behavior that the global controller. Overall blade aeroelastic stability margins were not significantly affected by application of HHC.

A global controller was used to compare the effects of HHC on roughly equivalent articulated and hingeless rotors at an advance

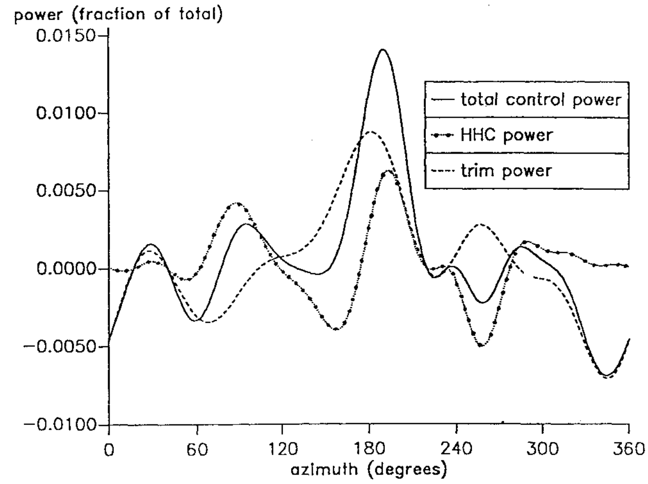

Fig. 4 Variation of instantaneous power, HHC power, and total control power with azimuth, articulated rotor, advance ratio $\mu = 0.030$, 5% blade root offset, and cautious global control.[20]

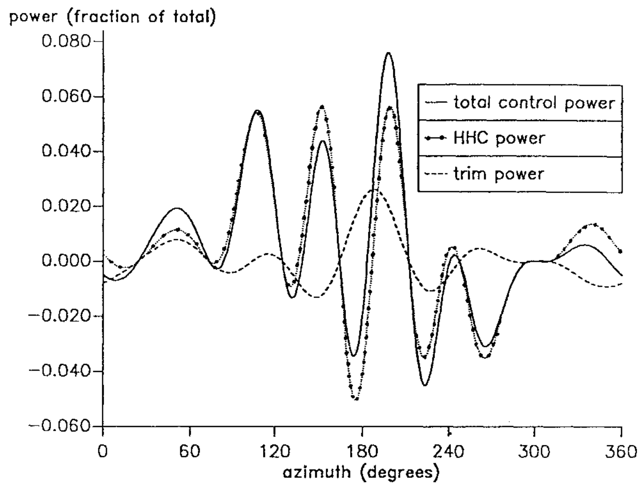

Fig. 5 Variation of instantaneous trim power, HHC power, and total control power with azimuth, hingeless rotor, advance ratio $\mu = 0.030$, 5% blade root offset, and cautious global control.[20]

ratio of $\mu = 0.30$. The conclusions obtained from this comparison were probably the most important item obtained in the course of this study. It was found that to implement vibration reduction on the hingeless rotor, significantly higher HHC angles were required. Substantial increases in blade root moments in the rotating system were observed for the hingeless rotor, and pitch link loads were also increased.

The instantaneous control power as a function of azimuth was calculated for both the hingeless and articulated rotor. This control power was calculated as the sum of the power required to produce 1/rev pitch changes needed to trim the helicopter, referred to as trim power, and the power required to introduce the HHC pitch changes, referred to as HHC power. Figures 4 and 5 show the variation in instantaneous trim power, HHC power, and total control power for the articulated and hingeless rotors, respectively. Note that Figs. 4 and 5 have different vertical scales. For the hingeless rotor the HHC power contributes relatively much more to the control power, as can be observed from the nature of these curves, which tend to follow one another closely (see Fig. 5). As evident from comparing the vertical scales, the maximum power excursions are on the order of five times larger for the hingeless rotor than for the articulated rotor and in addition are much sharper. The peaks are biased toward positive power values so that the total control power over one revolution is much higher for the hingeless than the articulated rotor. Application of HHC to the hingeless rotor led to an increase in required power of 1.4% whereas for the articulated rotor this increase was only 0.2%. Thus it appears that implementation of HHC on a hingeless

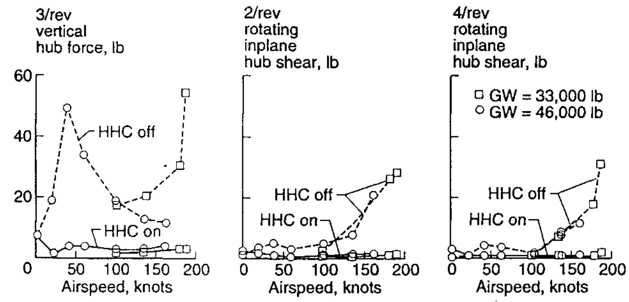

Fig. 6 Measured HHC effectiveness in trimmed level flight.[22]

rotor would require more powerful actuators and stronger control linkages than for an articulated rotor.

A similar simulation capability was also developed by Nguyen and Chopra.[29,30] The structural model in these studies is similar to that in Refs. 20 and 21. However, the aerodynamic model is more refined since it includes both nonlinear time-domain aerodynamics combined with an induced inflow calculation based on a free wake model. The HHC algorithm used was identical to that used in Ref. 20. This HHC simulation capability was validated by comparing it with wind tunnel tests conducted on a one-sixth dynamically scaled three-bladed articulated rotor model tested by Boeing Helicopter Company[22] up to advance ratios of $\mu = 0.40$, and fairly good correlation with the experimental data was obtained.

A very comprehensive wind tunnel test program on a three-bladed articulated model rotor (CH-47D) was also conducted by Shaw et al.[22] It was found that HHC is highly effective on articulated three-bladed rotors. Harmonic hub load response to HHC was found to be linear up to 3 deg and it was insensitive to flight condition. Furthermore, a fixed-parameter control law was found to be fully effective for vibration reduction. Figure 6, taken from Ref. 22, shows that closed-loop HHC was extremely effective in suppressing vibratory hub forces. The fixed-gain controller, when configured to regulate 3/rev vertical force and 2/rev and 4/rev inplane shears, suppressed all three of these components simultaneously by 90% in almost all of the trimmed and quasi-steady maneuvering test envelope. In addition to numerous important conclusions, Ref. 22 also contains an excellent concise review of the state of the art of vibration reduction using HHC.

Another important body of research dealing with the application of HHC to a four-bladed hingeless rotor can be found in Refs. 31–34. In Ref. 32 fairly extensive tests on a four-bladed hingeless rotor with a minimum-variance controller are described by Lehmann and Kube. An adaptive local controller was found to perform quite well over the whole envelope tested. Effective vibration reduction was also obtained with a gain adjustment (adaptation) algorithm. The power required for HHC during the test was not considered in this study. Very interesting additional tests together with simulation results were presented in a follow-up study.[33] It was found that a 3/rev controller can produce simultaneous reduction of all the vibratory components, and the resulting vibration level is similar to that achievable by a combination of 3/rev, 4/rev, and 5/rev HHC combination. Furthermore, it was noted that a partial adaptive controller performed as well as the full adaptive controller.

In a more recent study[34] a fixed-gain controller has been developed that produces vibration reduction comparable to that obtained in the previous studies, which utilized the adaptive controllers.

A recent paper[35] has also suggested that adaptation may not be required to accomplish vibration reduction using HHC; however the mathematical model of the helicopter used to reach this conclusion was too simplistic.

Another approach to the vibration reduction problem using HHC has been pursued by Gupta,[36] Gupta and DuVal,[37] and Duval, Gregory, and Gupta[38]; they have developed an extension of linear quadratic Gaussian (LQG) design methods using frequency-shaped costs functionals. A vibration controller was obtained by minimizing a cost functional that places a large penalty on fuselage accelerations at set vibration frequencies. The optimal control solution involved feedback of fuselage accelerations through undamped oscillators tuned to the frequency at which vibrations are to be suppressed. This approach has the advantage that on-line harmonic analysis of the vibrations is not required and the resultant controller is simple to implement because it is a constant-gain regulator with filters in the feedback loops. A dynamic model of the rotor/fuselage combination is needed in the LQG design procedure. Since this model will change with flight condition, gain scheduling is required to account for these different conditions. This control law was implemented using a blade element simulation of the Rotor Systems Research Aircraft (RSRA). Accelerations were reduced by at least 80% in all channels except the vertical, in which the initial vibration level was two orders of magnitude below that of other channels.

From the discussion presented in this section it is evident that the research conducted in the last decade has produced remarkable advances in this field. It is also clear that comprehensive aeroelastic simulations of HHC play a very useful role in this field because they are more cost effective than wind tunnel tests, provide improved fundamental understanding of the underlying phenomena, and serve as an ideal basis for planning such tests. Recent results also imply that implementation of HHC on hingeless rotors could be more difficult and less effective than similar implementation of HHC on articulated rotors. Finally it appears that adaptive controllers may not always be required in order to implement effective HHC for vibration reduction.

Based on the material presented in this section it is evident that despite the demonstrated feasibility of HHC and the relative maturity of this technology it has not been implemented on an actual production helicopter. One can find several reasons for this situation: 1) the considerable cost for implementing HHC on a production helicopter is quite high, 2) limitations on the objectives that can be achieved with HHC implemented through a conventional swashplate, and 3) questions regarding the effectiveness of HHC for vibration reduction in hingeless and bearingless rotors.

III. Individual Blade Control

An alternative to control through a conventional swashplate is the IBC approach in which each blade is individually controlled in the rotating reference frame over a wide range of frequencies. This control concept, which was pioneered by Kretz,[39] is a more general approach that removes some of the limitations that exist on active control through a conventional swashplate. IBC involves, not just control of each blade independently, but also a feedback loop for each blade in the rotating frame.

A considerable amount of research in this area has been done by Ham and his associates; this body of research was summarized in Ref. 40. For certain applications where dynamic phenomena occur at harmonics of rotor rotational speed, IBC can also be implemented through a conventional swashplate, provided that one has a three-bladed rotor.

The research described in Ref. 40 used a simple wind tunnel model combined with the concept of modal control. A number of important applications of IBC were considered. Among these applications some of the more interesting ones were. 1) gust alleviation, attitude stabilization, and vibration alleviation[41]; 2) lag damping augmentation[42]; 3) stall flutter suppression[43]; and 4) flapping stabilization in forward flight.[44] It is evident from these applications that the additional freedom provided by IBC can be used to provide alleviation of undesirable dynamic phenomena that goes beyond vibration reduction.

Practical implementation of IBC has been recently considered in the helicopter industry.[45] The system described in Ref. 45 uses actuators located between the swashplate and the rotor blade. It is based upon the idea of superimposing IBC upon the 1/rev swashplate control. The system was tested on the whirl tower, and it is implemented for the MBB BO-105 hingeless rotor system. This system was also successfully flight tested in 1990 and 1991 at Eurocopter's Ottobrun facility with low imposed control authorities of ± 0.19 deg and ± 0.42 deg, respectively.[45] This system was also recently tested in the 40×80-ft wind tunnel at NASA Ames Research Center.[46]

From this relatively brief discussion it is clear that IBC has not reached a level of maturity comparable to HHC. However, its implementation on a production helicopter may require the replacement of the conventional swashplate by its "electronic" counterpart. Such a drastic step could only be justified if the advantages of such a system could be demonstrated in an unequivocal manner. To reach such a stage a substantial amount of additional research on IBC is needed.

IV. Vibration Reduction Using an Actively Controlled Flap

Both HHC and IBC are based upon the idea of oscillating the whole blade at its root with fairly high frequencies. Modern helicopters have been evolving toward hingeless and bearingless rotor configurations. Therefore, implementation of HHC or IBC for these types of rotor configurations might require considerable power, since one has to oscillate a highly coupled, flexible, structural dynamic system at relatively high frequencies.[20] It is therefore logical to examine the feasibility of vibration reduction in helicopter rotors using an alternative concept based on an actively controlled trailing-edge flap located on the blade. Advantages of this approach are low power consumption and enhanced airworthiness, since the control system employed for vibration reduction is independent of the primary control system, which uses the conventional swashplate. It should be noted that this concept is not entirely new. Over 20 years ago Lemnios and Smith[47] used a servo flap in the context of their research on the controllable twist rotor (CTR). Using a combination of collective and cyclically varying twist distributions on the blade, they demonstrated a considerable increase in performance and a 30% decrease in blade-bending amplitudes.

In two recent papers Millott and Friedmann[48,49] have conducted a fairly comprehensive study of vibration reduction using an actively controlled trailing-edge flap. This study was conducted in two stages; The first stage[48] was based on the simple offset, hinged spring, restrained blade model, with coupled flap-lag–torsional dynamics, and a partial span, actively controlled, trailing-edge flap, shown in Fig. 7. The objective of the first stage was to demonstrate the feasibility of this concept. The second stage[49] was based on a fully flexible blade model, with an actively controlled flap shown in Fig. 8. The emphasis in this portion of the study was on the practical implementation of this novel vibration reduction concept.

A brief description of this research and the most interesting results obtained in Refs. 48 and 49 are given below. Each blade of the four-bladed hingeless rotor has a partial span, trailing-edge flap, which is used to introduce control input directly in the rotating reference frame, given by

$$\delta(\psi_k) = \sum_{n=2}^{N_{c,\max}} [\delta_{nc}\cos(n\psi_k) + \delta_{ns}\sin(n\psi_k)] \tag{7}$$

In both studies, the nonlinear equations of motion[48,49] were derived employing an explicit formulation using MACSYMA. The inertial loads, based on D'Alembert's principle, contain the inertia terms due to the dynamics of the actively controlled flap.

The aerodynamic loads are based on quasi-steady Greenberg theory, including the effect of the actively controlled flap. An empirical hinge moment correction factor is also used, and reverse flow effects are included. In this context it is important to mention that the effect of unsteadiness on this particular class of problem is fairly modest. The effect of unsteady Greenberg theory using augmented states and

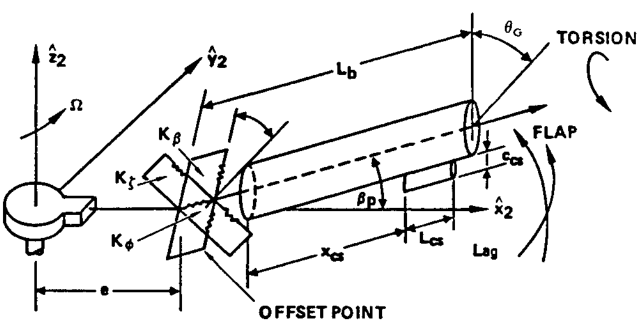

Fig. 7 Offset, hinged spring, restrained blade model with actively controlled trailing-edge flap.

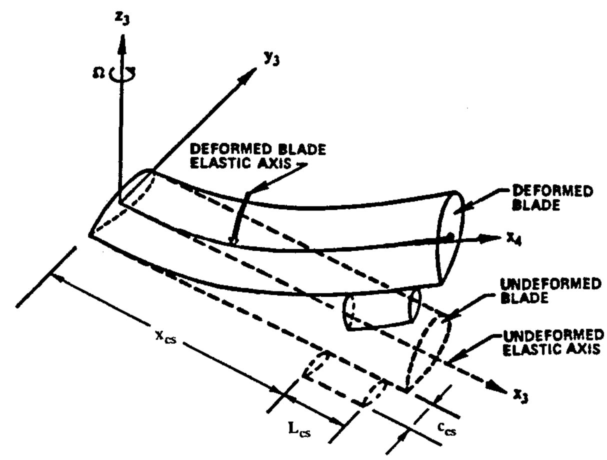

Fig. 8 Fully flexible blade model with actively controlled trailing-edge flap.

time-domain aerodynamics has been considered in Refs. 20, 21, 28, and 59 for the conventional HHC studies conducted in Refs. 20, 21, and 28. It was found that the primary influence was on the torsional response, and the difference betwen the quasisteady and unsteady blade response was less than 15% for all the cases considered. Probably, improved modeling of the unsteady compressibility and dynamic stall could be significant, and therefore ongoing research (at the University of California, Los Angeles) is aimed at determining the influence of these effects on the performance of the actively controlled flap. On the other hand, low-frequency wake aerodynamics, such as dynamic inflow, is expected to have a minor influence on the performance of the actively controlled flap.

The propulsive trim equations, satisfying longitudinal and vertical force equilibrium as well as pitch and roll moment equilibrium, are solved simultaneously with the blade equations of motion so as to produce a completely coupled trim/aeroelastic analysis. The solution to these coupled equations is obtained using the harmonic balance technique. The steady-state response of the blade is obtained from the solution of a system of coupled nonlinear algebraic equations.

Blade loads in the rotating system are obtained by summing the inertia and aerodynamic loads over the blade span. Subsequently these loads are transformed to the hub fixed system and summed over the four blades to yield the 4/rev vibratory hub shears and moments.

The optimal control algorithm is based on the minimization of a quadratic cost functional containing the squares of the vibratory hub loads and control activity:

$$J_i = \{Z_i\}^T[W_Z]\{Z_i\} + \{u_i\}^T[W_u]\{u_i\} + \{\Delta u_i\}^T[W_{\Delta u}]\{\Delta u_i\} \tag{8}$$

A deterministic, discrete-time controller is obtained by minimizing J_i with respect to the control $\{u_i\}$ to yield the optimal control $\{u_i^*\}$.

It is important to emphasize that this approach is quite similar to the approach used in conventional HHC, described in Sec. II, and it is based on the linear quasi-static frequency-domain representation of rotor response to control. Like in the case of conventional HHC, both a global controller and a local controller (see Sec. II) were implemented.

The power required to drive the control flap on the kth blade is averaged over one revolution and multiplied by the number of blades to produce a measure of the power required to implement the control:

$$P_{cs} = \sum_{n=1}^{N_b=4} \frac{1}{2\pi} \int_0^{2\pi} M_{HCS}(\psi_k)\dot{\delta}(\psi_k)\,\mathrm{d}\psi_k \tag{9}$$

The results obtained in the first stage[48] are for a 20% span flap, centered about the 75% span position and a chord equal to 0.25 of the blade chord. All the results were calculated for an advance ratio of $\mu = 0.30$. The blade was a soft-in-plane blade configuration, with

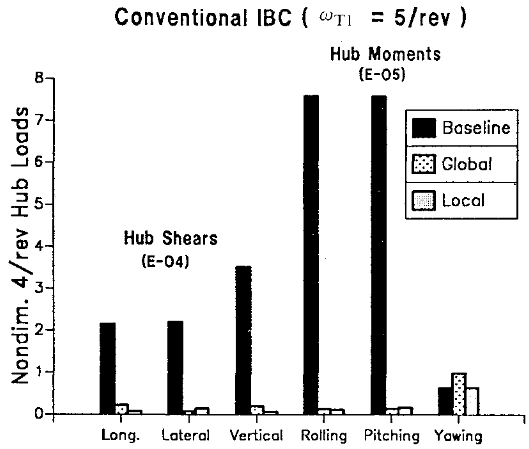

Fig. 9 Four-per-revolution hub shear and moment reduction for conventional IBC, torsionally stiff blade.

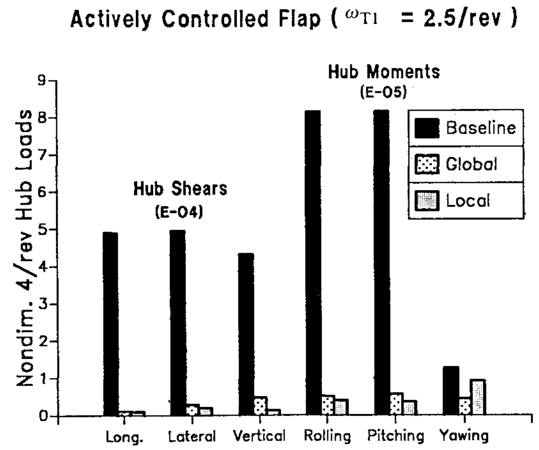

Fig. 11 Four-per-revolution hub shear and moment reduction using actively controlled flap, torsionally soft blade.

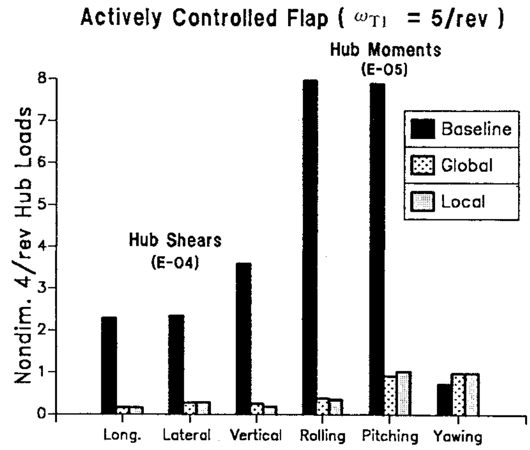

Fig. 10 Four-per-revolution hub shear and moment reduction using actively controlled flap, torsionally stiff blade.

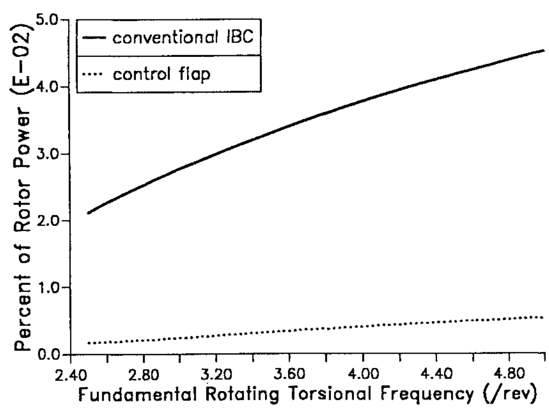

Fig. 12 Control power requirement for offset, hinged spring, restrained blade model; comparison of conventional IBC with ACF.

$\omega_{F1} = 1.15, \omega_{L1} = 0.57, 2.5 \leq \omega_{T1} \leq 5.0, C_W = 0.005, \gamma = 5.0, \sigma = 0.05$; and $W_u = W_{\Delta u} = 0$.

Input signals to the actively controlled flap contain a combination of 2/rev, 3/rev, 4/rev, and 5/rev harmonics in the rotating reference frame. The harmonic balance solution uses six harmonics to properly capture the effect of the 5/rev harmonic input.

Some typical results obtained in the first stage are described next. Figure 9 shows the vibration reduction for the 4/rev hub shears and moments obtained for conventional IBC when the blade is oscillated in its entirety on a torsionally stiff blade ($\omega_{T1} = 5.0$ /rev). Figure 10 depicts the reduction in hub shears and moments when using the actively controlled flap (ACF) on the torsionally stiff blade. Both figures present the baseline 4/rev components and their reduction. The vertical hub shear component is used as the representative indicator of the degree of vibration reduction achieved. The vertical hub shear was reduced to within 10% of its baseline value using an actively controlled flap, compared to a reduction to within 5% of its baseline value using conventional IBC. Thus, it is evident that the difference in the degree of vibration reduction achieved by the two control approaches is very small.

The degree of vibration reduction is influenced by the torsional frequency. Figure 11 shows the vibration reduction when using the actively controlled flap on the torsionally soft blade ($\omega_{T1} = 2.5$/rev). Comparison of Figs. 10 and 11 indicates that the performance of the actively controlled flap is somewhat better for the torsionally soft blade. Figure 12 depicts a comparison of the average power required (per revolution) for the implementation of vibration reduction using conventional IBC and the actively controlled flap. The power required, obtained from Eq. (9), is 8–10 times lower for the actively controlled flap, indicating that the actively controlled flap is a much more efficient vibration reduction device. Results presented in Ref. 48 also show that somewhat larger flap deflection, or control input, angles are needed when using the actively controlled flap when compared to the angles required by conventional IBC.

A substantial amount of more practical results were obtained in the second stage of this research.[49] These results are for a fully flexible blade represented by three flapwise, two in-plane, and two torsional modes. The actively controlled flap for this case was modeled as a 12% span, one-fourth chord, trailing-edge flap centered about the 75% span position. The blade fundamental frequencies are represented by $\omega_{F1} = 1.124, \omega_{L1} = 0.732$, and $2.5 \leq \omega_{T1} \leq 5.0$, other parameters being similar to stage 1.

The degree of reduction achieved in each of the 4/rev vibratory hub load components when the two control approaches are implemented on the flexible blade is shown in Figs. 13 and 14. The comparison is made for both for a blade relatively soft in torsion, $\omega_{T1} = 2.5$/rev shown in Fig. 13, as well as for a blade relatively stiff in torsion, $\omega_{T1} = 5.0$/rev presented in Fig. 14. The two figures demonstrate that both control approaches were successful in achieving very substantial reductions in each of the vibratory hub load components. In the case of the torsionally soft blade, the 4/rev hub shears and hub moments were reduced on average by 96 and 94%, respectively, using conventional IBC, and by 91 and 96%, respectively, using the actively controlled flap. Though the degree of reduction was slightly lower in the case of the torsionally stiff blade, in which case the 4/rev hub shears and hub moments were reduced on average by only 90 and 92%, respectively, using conventional IBC, and by only 78 and 81%, respectively, using the actively controlled flap, both approaches were judged to be quite effective in reducing the vibratory hub loads.

The power requirements for both blade models are presented in Fig. 15. The power estimates obtained using the flexible blade

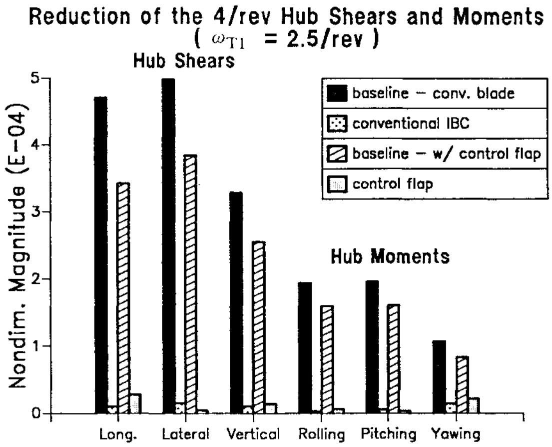

Fig. 13 Four-per-revolution vibration reduction with ACF on torsionally soft blade.

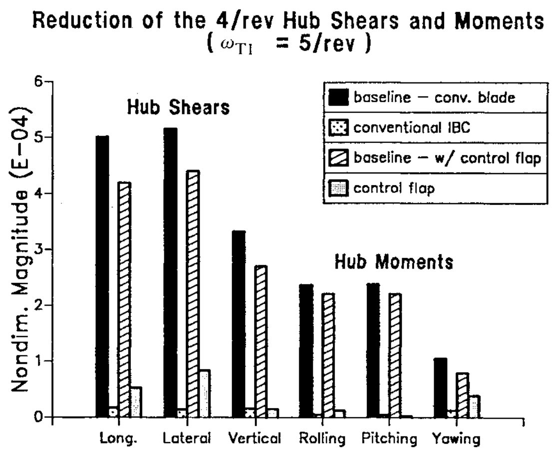

Fig. 14 Four-per-revolution vibration reduction with ACF on torsionally stiff blade.

model show that the actively controlled flap requires substantially less power than conventional IBC over the entire range of blade torsional frequencies; thus, between 10 and 20 times less power is consumed by the actively controlled flap when compared to the conventional IBC. It is also evident that the performance of the actively controlled flap for the elastic blade model, which represents a practical configuration, is significantly better than for the offset hinged spring restrained blade model.

Results not presented here[49] indicate that the control angles required for the actively controlled flap are practical (less than 8 deg) and the performance of the system is sensitive to the location of the control surface along the blade span.

Thus it can be said that the feasibility studies conducted on the actively controlled flap[48,49] have clearly demonstrated the remarkable potential of this device as a means for reducing vibrations in forward flight. Numerous additional results from this study are presented in a recent NASA report.[61]

An independent study[50] of the actively controlled flap has indicated that the practical implementation of such a device could be feasible using magnetostrictive actuation based on Terfenol-D. The estimated properties of the system would require approximately less than 1% of gross vehicle weight and use around 0.7% of cruise power. Another feasibility study conducted by Straub[51] has indicated that servo-flap control using hinged control surfaces driven by discrete actuators may be a suitable candidate for smart materials actuation based upon piezoelectric materials.

More recently the authors have also completed a theoretical study showing the feasibility of implementing the ACF concept using magnetostrictive actuation[62] similar to that studied in Ref. 50. The resulting actuator required only 630 g of Terfenol-D for the blade considered in Refs. 48 and 49. The total weight of the complete

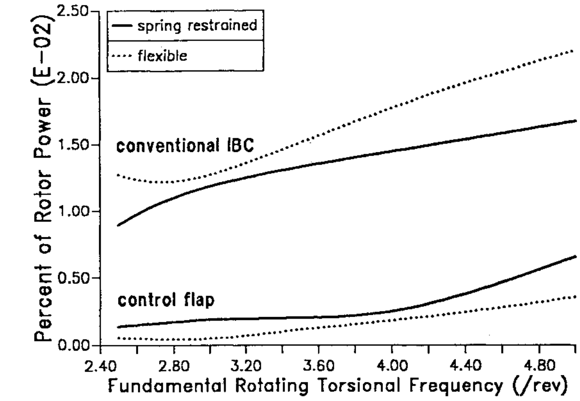

Fig. 15 Control power requirement comparison of conventional IBC with actively controlled flap.

vibration suppression system was estimated at less than 6% of blade weight.[52]

In a separate study conducted by Straub and Robinson[52] the feasibility of an actively controlled flap to reduce noise due to blade vortex interaction has been explored. These recent studies clearly reveal the considerable interest in exploring the substantial potential associated with this new active control concept.

V. Active Control of Structural Response

Recently, a new approach known as ACSR has been developed,[53] and it is discussed here briefly for the sake of completeness. In the ACSR approach fuselage vibration reduction is achieved by exciting the fuselage with actuators located between the rotor and the fuselage such that the sum of the response of the airframe, due to rotor loads and the external excitation, is reduced to a minimum.

The ground and flight tests performed on a Westland 30 four-bladed hingeless helicopter are described in Ref. 54. Four control actuators were mounted between the fuselage and the structure supporting the gear box. Twenty-four sensors located at various critical points were used for vibration measurements. During ground tests, the baseline vibration was simulated by applying appropriate moments at the rotor hub. The performance index minimized consisted of a combination of the weighted sum of measured vibrations and actuator activity. Using the open-loop transfer function for the control actuators, an 80% reduction from baseline vibration levels was achieved.

Flight tests on the Westland 30 with the ACSR system operating in the closed-loop mode at various flight conditions were also performed. The ACSR system provided remarkable vibration reduction throughout the whole flight envelope explored.[54]

Preliminary tests aimed at the implementation of an ACSR system on a Sikorsky S-76B helicopter have been conducted at Sikorsky,[55] and a substantial reduction in vibration levels containing all harmonics, including 1/rev, was observed.

The empirical and experimental work in this area indicates that the approach is quite promising; however, the actual mechanism of vibration reduction is not well understood, and additional research on this topic is needed.

VI. Coupled Rotor–Fuselage Modeling for Vibration Reduction

The various applications of vibration reduction using active control, discussed in the previous sections, are based on a frequently used assumption that implies that vibration reduction at the rotor hub is equivalent to vibration reduction at various locations in the fuselage. Thus it is important to examine the fundamental validity of such an assumption.

A number of studies[56–58] have addressed this question together with a number of related issues. The mathematical model for the coupled rotor/flexible fuselage based upon the idealized configuration shown is in Fig. 16. The rotor was represented by a four-bladed hingeless configuration with coupled flap-lag–torsional dynamics

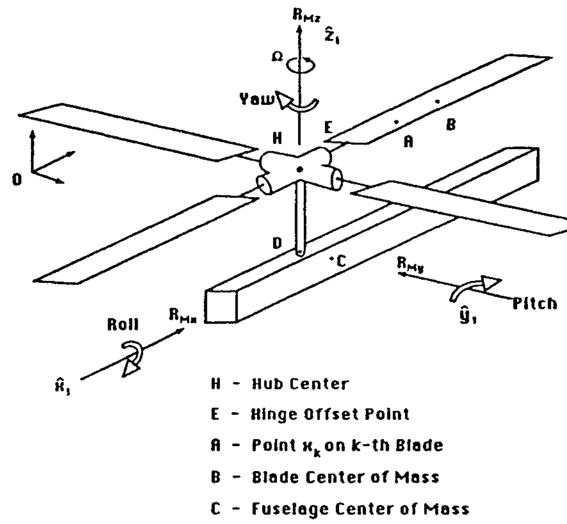

Fig. 16 Schematic representation of coupled rotor/flexible fuselage model.

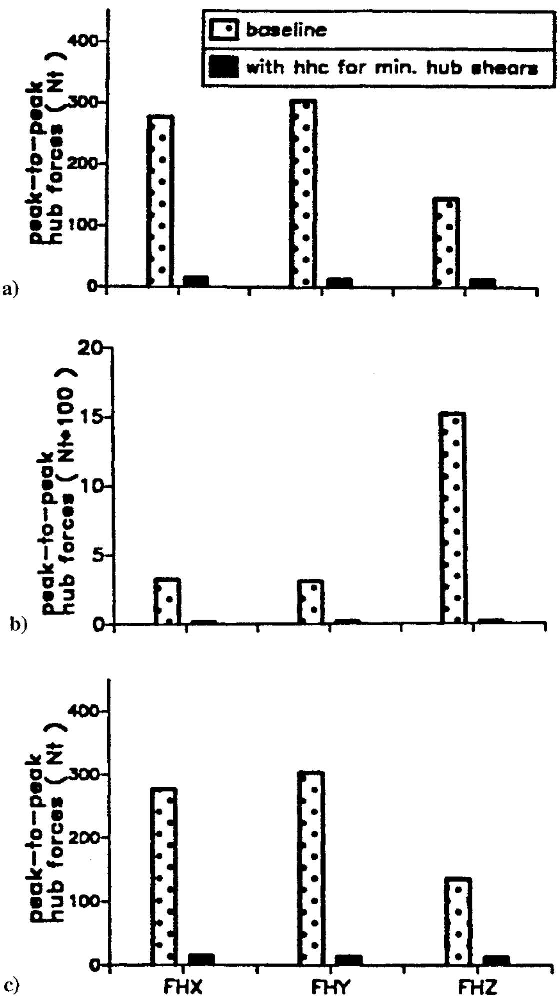

Fig. 17 Effect of HHC input for minimizing hub shears on hub shears: a) rigid fuselage, b) first fuselage bending frequency 4/rev, and c) first fuselage bending frequency 1/rev.

for each blade. The blades could be modeled using two alternatives: 1) an offset hinged spring, restrained blade model, as shown in Fig. 7, and 2) a fully elastic blade, as shown in Fig. 8. For both cases the blade model accounts for moderate deflections. The rotor was combined with a flexible fuselage modeled as a beam with five rigid-body degrees of freedom (no yaw) and six elastic degrees of freedom consisting of two vertical bending

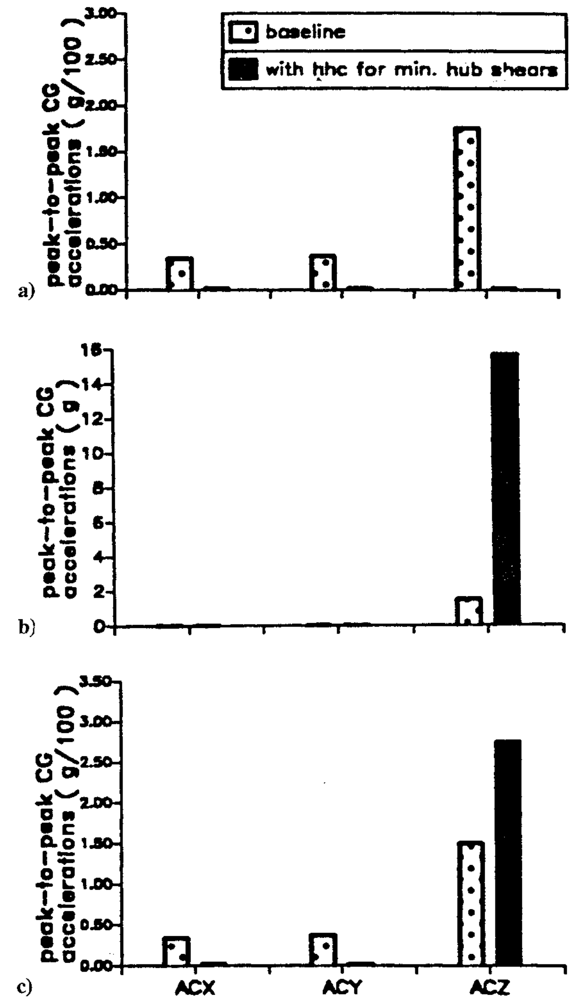

Fig. 18 Effect of HHC input for minimizing hub shears on fuselage CG accelerations: a) rigid fuselage, b) first fuselage bending frequency 4/rev, and c) first fuselage bending frequency 1/rev.

modes, two horizontal bending modes, and two torsional modes. The coupled rotor/fuselage equations were derived using a symbolic computing facility.[56]

The pitch input for the kth blade in the rotating system is given by

$$\theta_{HHk} = \left(\theta_{OC}^4 \cos 4\psi + \theta_{CS}^4 \sin 4\psi\right) + \left(\theta_{OC}^4 \cos 4\psi + \theta_{CS}^4 \sin 4\psi\right) \cos \psi_k + \left(\theta_{SC}^4 \cos 4\psi + \theta_{SS}^4 \sin 4\psi\right) \sin \psi_k \quad (10)$$

The coupled rotor/fuselage equations were solved using harmonic balance, and the trim and aeroelastic response solutions are fully coupled in forward flight.

Two cases were studied:

1) Minimize the 4/rev components of the hub shear, represented by

$$\{Z_F\} = \{F_{HX4C}, F_{HX4S}, F_{HY4C}, F_{HY4S}, F_{HZ4C}, F_{HZ4S}\}^T \quad (11)$$

2) Minimize the 4/rev components of fuselage acceleration at the center of gravity (CG), given by

$$\{Z_A\} = \{A_{CX4C}, A_{CY4S}, A_{CY4C}, A_{CY4S}, A_{CZ4C}, A_{CZ4S}\}^T \quad (12)$$

This was accomplished by using the global controller, represented by Eq. (4), in the open-loop mode. Based on this model, the HHC inputs required to minimize the hub shears or fuselage (CG) accelerations were obtained for various fuselage representations. Figures 17 and 18, taken from Ref. 58, show the peak-to-peak vibratory hub shears and fuselage CG accelerations, respectively, for the case when HHC is aimed at minimizing the hub shears. From Fig. 17 it can be seen that when HHC is aimed at minimizing the hub shears, the peak-to-peak vibratory hub shears are drastically reduced from their baseline values. In fact, the hub shears are practically suppressed for all fuselage representations shown in Fig. 17. Figure 18 shows the effect of HHC inputs aimed at hub shear minimization on the peak-to-peak fuselage CG accelerations. When the fuselage is treated as a flexible body (Figs. 18b and 18c) the vertical CG acceleration increases by a factor of 2–5 from its baseline value with the introduction of HHC input aimed at minimizing the hub shears. On the other hand, when the fuselage is treated as a rigid body (Fig. 18a), a reduction in the hub shears and CG accelerations occurs simultaneously. It should be noted that the fuselage CG accelerations are the three linear accelerations along the x, y, and z directions.

Additional results,[58] not shown here, indicate similar behavior when HHC for fuselage CG acceleration minimization is introduced. Again, when the fuselage is treated as a flexible body, vertical hub shears increase by a factor of 3–6 when HHC inputs aimed at minimizing accelerations are introduced. However, for the rigid-fuselage case accelerations and hub shears are reduced simultaneously. The case of simultaneous reduction of both hub shear and hub moments was also considered in Ref. 58. For the case of a flexible fuselage a reduction of approximately 50% in the vertical acceleration at the fuselage CG is obtained whereas the other components (lateral and longitudinal) remain virtually unchanged. This reduction in acceleration is accompanied by substantial reduction of hub moments and modest reduction of hub shears.

The important conclusion obtained from the few results presented here as well as the results presented in Refs. 56–58 is that cyclic inputs aimed at the reduction of hub loads (i.e., both shears and moments) may not lead to a simultaneous reduction in the accelerations at specific locations in a flexible fuselage.

VII. Concluding Remarks

Based on the review of several approaches to active control of vibration reduction in helicopter rotors, one can make a few observations that summarize the current state of this fascinating area of research.

1) Higher harmonic control has reached considerable maturity. However, its use in production of helicopters has not materialized, primarily due to excessive cost and certain limitations associated with its implementation through a conventional swashplate.

2) Individual blade control is fairly immature. However, implementation of this approach is encountering resistance due to reluctance to replace the conventional swashplate by its electronic equivalent.

3) The ACF is an effective means for achieving vibration reduction using practical input angles. The degree of vibration reduction achieved is comparable with conventional IBC. However, the power requirements are an order of magnitude less than for conventional IBC. The ACF has no effect on airworthiness, since it is independent of the primary control system. Additional research is needed to exploit the potential of this approach.

4) Preliminary studies indicate that implementation of the ACF may be possible using magnetostrictive or piezoelectric actuation.

5) Active control of structural response is another new approach for vibration reduction in the fuselage. Considerable research is needed to gain an improved fundamental understanding of this approach.

Acknowledgments

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